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NOTES AND EXERCISES

ON

SURVEYING

FOR THE USE OF STUDENTS IN

KENYON COLLEGE

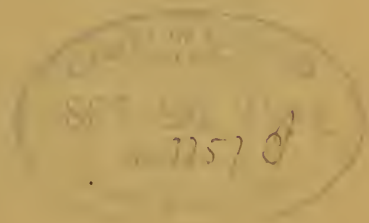
BY

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SURVEYING.

SURVEYING is that branch of applied mathematics which has for its object the measurement of the earth's surface, including heights, distances, and areas, and the representation of the same by a plat or chart.

Plane surveying is that in which small portions of the earth's surface are regarded as parts of a plane. *Topographical surveying* consists in the measurement and exact delineation of a place, including variations of level. *Geodetic surveying*, or Geodesy, relates to the whole earth, or to large portions of it, taking into consideration the curvature of the surface.

Surveying is to be distinguished, on the one hand, from mensuration, which is applied to the measurement of smaller objects, and on the other hand, from Astronomy, which relates to the sizes and distances of the heavenly bodies.

All measurement consists in two operations—observation and calculation. The results of observation are more or less imperfect, depending upon the keenness of the senses and manual skill. Calculation may be made to any required degree of accuracy.

An observation in surveying consists in measuring either a distance or an angle. These quantities, of both kinds, are observed by the direct application of a standard. In geodesy, however, as in astronomy, in addition to distances and angles, time is an object of direct observation.

NOTE.—This is not a treatise on Surveying. Many things are omitted which can be taught better orally, either in the class-room or in the field.

STANDARDS OF LENGTH AND OF ANGLE.

The standard measures of length are the meter and the yard. The meter is used in the geodetic surveys made by the government. The foot, divided decimally, is generally used by railway engineers.

The *meter*, when first adopted by the French, was intended to be one ten-millionth part of the distance on the surface of the earth from the equator to the pole. More accurate geodetic surveys have shown that the meter in actual use is a little shorter than this. As the meter has been adopted by nearly all enlightened nations as the primary unit for all measures, it is not probable that any effort will be made to correct the error.

The standard meter is an actual bar of platinum kept by the French government. The standard yard is an actual bar kept by the British government. The government of the United States has very accurate copies of these, but it is said that the American yard exceeds the British standard by nearly one-thousandth of an inch, and that there is a smaller error in the meter.

In all instrumental observation, the standard of angular quantity is the *degree*. The *grade* (the hundredth part of a right angle) is only used in some calculations.

INSTRUMENTS OF LENGTH.

In the geodetic surveys in the United States, bars of six meters length are used. The contacts are regulated by screws, and thermometers are inserted in the bars for observation of the temperature of the metal.

The common *surveyor's chain* is four rods long, in one hundred links, but the instrument generally used is a half-chain. Railway engineers use a chain of one hundred feet.

When the common surveyor's chain is used for measuring horizontal distance, the two ends must be held at the same level, whatever be the actual unevenness of the surface. Hence, it is necessary, in testing the chain, to allow for the diminution in length caused by the sagging when suspended. It also becomes the rule that all chaining is done by the chain held up. Evidently some skill is required to make uniform and correct measures of distance in this way. Sometimes, on a steep hill-side, it is necessary to use a smaller part of the chain.

In using the eleven pins with the chain, observe these rules:—

1. One pin is put at the beginning of the distance to be measured.

2. The chainman in advance takes ten pins. 3. The pin in advance is fixed in its place *before* the pin at the rear is taken up, so that one pin is in place all the time.

The following errors in chaining should be avoided:

1. The rear chainman errs in not holding the end of the chain either against the pin or exactly over it.

2. The forward chainman errs in not sticking the pin vertically under the end of the chain. If the chain is more than a foot from the ground, the pin must be dropped.

3. Sticking a pin out of the line. The rear chainman should keep in view the signal at the end of the line; and should correct the position of the other chainman if he is right or left of the signal.

4. Counting half-chains as chains.

5. Counting a chain or half-chain too many. Let the last pin before the end of the line remain in the ground till the distance is noted; then the number of pins in the rear chainman's hand shows the number of lengths of the chain or half-chain.

6. Writing the number of links, when less than ten, as tenths of a chain instead of hundredths.

Skill in the use of these and all other instruments can be acquired only by practice, under the direction of a competent surveyor.

Exercise.—Let a line be measured both by chain and by rod; let this be done by various members of the class. Compare the results, and consider the probable limits of error.

The *levelling staff*, or *rod*, is used for observing differences of level, that is, vertical distance. In its use, it must be held as nearly vertical as possible.

In the *levelling instrument*, the parts to be noticed are:—

1. The tripod;
2. The joint, and levelling screws;
3. The vertical axis, revolving bar and wyes;
4. The telescope and spirit level;
5. The adjustment of the telescope parallel to the spirit level;
6. The adjustment of the telescope perpendicular to the vertical axis.

INSTRUMENTS OF DIRECTION.

These are the compass, the transit, the theodolite, and the sextant.

A horizontal angle is one having its arms in a horizontal plane. A vertical angle is one having its arms in a vertical plane, one of the

arms being horizontal. It is an angle of elevation or of depression, according as the other arm is above or below the horizon.

The *compass* is used for measuring the horizontal angle which any line makes with the meridian. The *engineer's transit* is used for measuring any horizontal angle. The *theodolite* is used for measuring both horizontal and vertical angles. The *sextant* is used for measuring angles in any plane whatever.

In the compass, the parts to be noticed, are: 1. The staff; 2. The joint; 3. The box; 4. The circle; 5. The needle; and 6. The sights. Some compasses have a spirit level. The solar compass has an apparatus for determining the true meridian by observations of the sun.

In the transit there are to be noticed:—

1. The tripod and the plummet;
2. The joint and levelling screws;
3. The vertical axis with its clamp;
4. The graduated circle and vernier plate;
5. The spirit level and the telescope.

The theodolite has, besides the above, a vertical graduated circle and a vernier.

The *vernier* is an ingenious contrivance for more accurate measurement. It is applied to the graduated standard of measure, to the graduated arc in angular measurement, and to the graduated bar or rod in linear measurement.

The vernier moves along the principal scale, and is so divided that n divisions of it are equal to $n-1$ or $n+1$ of the smallest divisions of the scale. The zero point of the vernier indicates the position on the scale which is to be ascertained. If this exactly coincides with a dividing mark, then the measure is read on the principal scale, and no reading of the vernier is required; but when the zero point of the vernier comes between two marks on the scale, the main scale is first read and there must be added an amount ascertained by the vernier. The amount to be added is indicated by that division mark on the vernier which exactly coincides with one on the scale. If this mark is the m th from zero, there must be added m times the n th part of the quantity indicated by one of the smallest divisions on the principal scale. This n th part is called *the least count* of the vernier.

The explanation of the principle of the vernier is left to the student, an easy task when the instrument is in his hands. (In the theodolite of K. C., n is 30, and the least count is one minute of arc. In the levelling rod, n is 10, and the least count is one-thousandth of a foot.)

The following errors in observing angles should be avoided:

- (a) With the compass—1. Reading East for West, and *vice versa*;
2. Reading the angle when the sights are not on the signal; and 3. Having the compass out of level.

- (b) With the theodolite—1. Neglect to notice in which way the vernier plate is turned, and whether the zero mark is passed over;
2. Not having the hair line on the signal when the angle is read;
3. Not having the lower telescope* on its mark; and 4. Allowing the instrument to be out of level.

- (c.) With all instruments—Error of parallax in reading grades; i. e. holding the eye to one side.

Exercises:—1. Make a drawing and a description of each instrument used by the class, specifying the use and purpose of the several parts.

2. Measure with the theodolite, the three angles of a triangle. The difference between the sum and two right angles shows the amount of error. Consider this knowledge of total error and compare this work with that of measuring a line.

3. Measure with chain and theodolite, two sides and the included angle of a triangle; calculate the third side by trigonometry, and then measure it by chain. Consider in which of the measured elements was probably the greater source of error, taking into view all the circumstances.

Students should be cautioned not to abuse instruments. Treat every instrument gently. Put no strain on a screw. If a grain of sand gets into a joint, let the instrument be taken apart and cleaned; do not use a dirty joint. A chain should not be tangled. No more force than is needed to lift the links should be used in untangling a chain.

INSTRUMENTS FOR PLATTING.

Every student should be provided with a scale, a protractor, and a pair of dividers.

Exercises.—1. Draw any triangle on paper; measure each angle with the protractor. The difference between the sum and two right angles shows the amount of error in measurement.

2. Measure two sides; from these and the angles calculate the third side; then measure and compare. This exercise may be varied to suit every case of equality of triangles.

3. Make plats of work done in the field.

HEIGHTS AND DISTANCES.

The solution of the principal problems of this class, by triangulation, is explained in the Trigonometry, Art. 875.

*Many theodolites do not have the lower telescope.

In applying these methods in the field, the student should endeavor to measure every line in at least two independent ways.

LEVELLING.

When great accuracy is required, as in mining and making railways and canals, differences of level are measured by the levelling instrument.

By setting the instrument at nearly equal distances from the preceding and the following station, the observer may avoid the effect of a possible maladjustment of the instrument. Some authors advise that this be done in order to avoid the error arising from the curvature of the earth's surface. A little observation and calculation will show that no ordinary instrument is sufficiently accurate to detect any error that could be attributed to this cause. At the distance of 200 feet the deflection is less than one-thousandth of a foot.

When the object of levelling is simply to ascertain the difference in level of two points, and no plat is to be made, then the notes need record only the "back sights and fore sights" in two columns. Thus, to find the height of Rosse porch above Ascension door-sill:

	<i>Back Sights.</i>	<i>Fore Sights.</i>
1st.....	3.496 feet.	2.351 feet.
2d.....	9.620	0.705
3d....	10.721	4.388
	<hr/>	<hr/>
	23.837	7.444
	7.444	
	<hr/>	
	16.393 feet.	

When, however, a profile is to be made, as of a road or canal, the level at every 100 feet, or other certain distance, should be recorded. The profile usually has a larger scale, ten-fold or more, for the heights than for the horizontal distances.

In a topographical map, contour lines are made; i. e., lines showing where level planes would intersect the actual surface, these planes being at regular intervals of height above some established base. The proper location of such lines is found by levelling, the details of the work depending upon the peculiarities of each locality.

Exercises.—1. Ascertain the height of a hill by levelling over two different routes. Compare, and discuss the probable error.

2. Measure then the same height by triangulation, by at least two sets of observations and calculations. Compare as before.

3. Determine the relative degrees of accuracy of the two methods.

AREA OF LAND.

In determining the area of land, the inequalities of the surface are disregarded, the area to be measured is that of the horizontal plane within the boundaries. The length of a side is the horizontal distance between its extremities. The angles are horizontal.

When a piece of land is in the shape of a square, a rectangle, a parallelogram, a triangle, a trapezoid, or any regular geometrical figure, the method of measuring its area consists of applying the geometrical principle.

When the shape is that of a polygon, that is, when all the sides are straight, the usual method of surveying is by a system of triangles and trapezoids. This method is described in the following pages. When one of the boundaries is an irregular curve, as the bank of a stream, the error may be diminished at will by substituting straight lines nearly coincident with the curve. Sometimes it is more convenient to divide such a tract, surveying the more irregular part by itself. The advantage of this is in excluding many small sides from the calculation of the whole area.

The *field notes* of a survey are the record, made on the spot, of the bearing and distance of each side of the field. For example, the following are the field notes of the survey of *Jan's Lot*, a six-sided field:

	<i>Bearings.</i>	<i>Distances.</i>
1.	S. 89° E.	5.335
2.	N. 26° E.	1.60
3.	N. 55° W.	4.57
4.	S. 20° W.	1.842
5.	S. 87° W.	1.593
6.	S. 2° W.	2.15

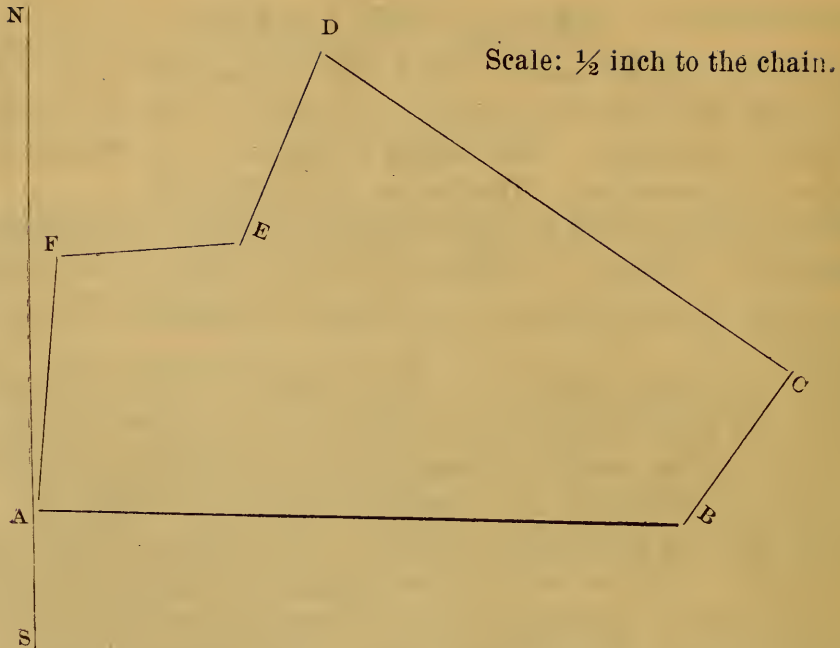
The *bearing* of a line is the angle, not over 90°, which the line makes with the meridian, the letters indicating the quarter of the compass.

The *distance* of a line or side of a field, is its length.

The degree of accuracy required depends entirely upon circumstances. Only very valuable land need be measured to a tenth of a link.

The method of calculating the area is best explained by means of a plat. In practice, the plat may be made either before or after the calculation.

First, make a line parallel to the side of the paper, for a prime meridian. This line should pass through either the easternmost or the westernmost corner of the plat. In the following plat, which illustrates the given field notes, the meridian NS passes through the western corner.



With the protractor, make the angle $BAS = 89^\circ$, that being the first bearing. Make the length of the side AB according to the scale. Since the direction of the next side is 26° East of North, and the direction BA is 89° West of North, the angle ABC must be made 115° . Since the next direction is 55° West of North, and CB is 26° West of South, the angle BCD must be made 99° , that is $180^\circ - (55^\circ + 26^\circ)$. So on, for the angles D, E, and F. The angle at each corner depends upon the preceding and the following bearing. It may be, 1. the sum of the bearings, or 2. the supplement of that sum, or 3. the difference of the bearings, or 4. the supplement of that difference. If the survey and drawing are accurate, the end of the last side falls upon the initial point A.

When it is important to make an exact plate, it is better to draw a meridian line at every corner. The method given in the last paragraph renews at every angle the errors made at all the previous angles. A very accurate plat can be made, after calculating the latitudes and departures, by using them to lay off the sides of the field.

From B, C, D, E, and F, let the perpendiculars BG, CH, DJ, EK, and FL, fall on the prime meridian NS. (These are not given in the diagram, but should be made by the student.) These perpendiculars form the bases of four trapezoids and two triangles. The other sides of these six figures are either sides of the field or parts of the prime meridian. From the observed bearings and distances, we may calcu-

late the bases and altitudes, and thence the areas of all these figures; then subtracting the sum of the four areas which are outside of the field from the sum of the other two, which contain these four *and* the field, we have the area of the field. See the calculation.

Calculation of the Area.

Latitudes.		Departures.		Correct's		Balanced.		D.M.D.	Double Areas		
North.	South.	East.	West.	L.	D.	Lat.	Dep's.		North.	South	
1		.093	5.334		+2	+4	.095	+5.338	5.338		.507
2	1.438		.701			+1	1.438	+ .702	11.378	16.362	
3	2.621			3.744	—1	—3	2.620	—3.741	8.339	21.848	
4		1.731		.636		—1	1.731	— .635	3.963		6.860
5		.083		1.591		—1	.083	—1.590	1.738		.144
6		2.149		.075		—1	2.149	— .074	.074		.159
4.059		4.056	6.035	6.046						38.210	7. 70
											7.670
Error		3	Error	11							
										20)	30.54
											1.527

The *latitude* of a side is the distance which one end is North or South of the other. Thus AG is the latitude of the side AB; HG is the latitude of BC, etc. The latitude of a side is its projection on a meridian line. It is the product of the distance by the cosine of the bearing.

The *departure* of a line is the the distance which one end is East or West of the other. Thus GB is the departure of the side AB; HC—GB is the departure of BC, etc. The departure of a side is its projection on a parallel of latitude. Also, it is the product of the distance by the sine of the bearing.

The latitudes and departures are ascertained by aid of a *traverse table*. This is a table of the latitudes and departures when the distance is unity, at every quarter of a degree of the quadrant. The traverse table used by professional surveyors gives the latitudes and departures with the multiples for the first nine digits. The one on pages 15 and 16, is sufficient for learning the principle.

The latitudes and departures are distinguished as *Northings*, *Southings*, *Eastings*, or *Westings*, according to the bearings, and are placed in four separate columns. If the survey and calculations are accurate, the sum of the northings must be equal to the southings, and the sum of the eastings to that of the westings. Equality in these is not to be expected, for the observations are not perfect, the angles being rarely

measured to less than one-fourth of a degree, or the sides to less than one link. Whether the error may be disregarded is a question of economy, which depends upon the amount of the error and the value of the land. If the error is not too great, it should be distributed, so as to make the northings equal to the southings, and the eastings equal to the westings.

When there is no reason to suspect that the error belongs more to one side than to an other, it may be distributed in proportion to the distance. For example, in the above survey, the sum of the distances is 17 chains and 9 links; the error of latitude 3, and the error of departure 11, may be distributed as in the two columns of *corrections*. Notice that as the southings are less than the northings, a correction applied to a southing is marked +, and one applied to a northing —. The same principle is used in the signs of the corrections of departures.

This mode of distributing the error is well enough under the circumstances stated, but the surveyor who has been over the whole line with chain and compass usually knows that the error is probably due to the sides, in a different ratio from that of their lengths. On a short side there may be many obstructions and difficulties, none of which are met on a long side. The skilful surveyor distributes the error according to his judgment of all the circumstances.

The figures in the columns of *balanced* latitudes and departures are found by applying the corrections to the previous columns. They are “balanced” because the sum of the northings equals that of the southings, etc.

The balanced departures of those sides which tend or depart *from* the prime meridian are marked +, and those which tend *towards* it —. When the prime meridian is on the east, the westings are positive and the eastings are negative.

The *double meridian distance* of a side is twice the distance of the middle point of that side from the prime meridian.

In the case of the first side, the double meridian distance is equal to the departure, as appears by the plat. The same is true for the last side. For each of these two, the double meridian distance is the base of the triangle; for every other side, it is the sum of the bases of the trapezoid.

The double meridian distance of any ~~side~~ ^{side} is found by the following

Rule.—Add to the double meridian distance of the preceding side the sum of the departures of both. The algebraic sum is intended, that is, a negative departure must be subtracted.

The proof of this rule is the geometric truth: the middle point of any side is farther (algebraically) from the prime meridian than the middle point of the preceding side, by half the sum of the departures of the two sides. Thus, the middle point of BC is farther from NS than the middle point of AB, by half the sum of the departures of AB and BC. Likewise, the middle point of CD, etc., but notice that the departure of CD is negative.

If the departure of the last side is not equal to the double meridian distance found by this rule, there is some error in the work.

Now we have the altitudes and the bases of all the triangles and trapezoids—the altitudes being the same as the latitudes, and the bases or sum of the bases in each case being the double meridian distance.

The product of the base, or the sum of the bases, by the altitude, gives the *double area*. Subtracting the sum of the exterior areas from the sum of those which include the whole figure, (in this case the South areas from the North) the remainder is twice the area of the field, expressed in square chains. Dividing this by 20 reduces it to acres.

It may be that the first station given in the field notes is not either the easternmost or the westernmost corner of the field. By an inspection of the departures the surveyor sees which are these two corners, and he begins the calculation of the D.M.D. at one of them. If he begins at any other corner, at least one of the double meridian distances is negative, which produces a needless difficulty in the calculation.

As a general rule, the latitudes and departures should be calculated to one more decimal place than is used in the field notes.

Exercises.—1. Make a plat and calculate the area of the lot having this boundary: Beginning at a certain corner, thence South $8\frac{1}{4}$ degrees East 22 chains and 81 links; thence North 53 deg. West 23 chains and 21 links; thence North $12\frac{1}{2}$ deg., West 13 chains and 20 links; thence North 65 deg., East 17 chains and 82 links; thence South $9\frac{1}{2}$ deg., East 12 chains to the place of the beginning.

2. Divide this lot into halves by a line from the corner where the field notes begin.

3. Run an East and West line through this lot, leaving two acres on the North side.

4. The field notes of one side of a four-sided field are lost; the remaining three are:

North 24°	West 34.	chains
North 50°	East 28.34	"
South 4°	East 34.20	"

Supposing these to be correct, what are the 4th bearing and distance?

5. Find the number of hectares within the following boundary :

1	S.	$58^{\circ}\frac{1}{2}$	W.	292.7	meters.
2	N.	34°	W.	198	"
3	N.	$81^{\circ}\frac{1}{4}$	W.	212.4	"
4	N.	$36^{\circ}\frac{1}{4}$	E.	247.3	"
5	N.	$7^{\circ}\frac{3}{4}$	E.	115.8	"
6	N.	$79^{\circ}\frac{1}{2}$	E.	154.	"
7	S.	$86^{\circ}\frac{3}{4}$	E.	205.5	"
8	S.	$12^{\circ}\frac{1}{4}$	W.	181.5	"
9	S.	25°	E.	219.2	"

6. After surveying a field, it was discovered that the half-chain used was 33 feet 4 inches long ; what correction was necessary to determine the true area ?

7. Calculate this : From beginning N. 10° W. 18 chains ; thence N. 20° E. 20 chains ; thence S. 89° E. 30 chains ; thence S. 9° E. $36\frac{1}{2}$ chains ; thence W. 39 chains to place of beginning.

TRAVERSE TABLE.

Bea'g	Latitude	Depart'e		Bea'g	Latitude	Depart'e	
0			0	0			0
$\frac{1}{4}$	1.0000	0.0044	$\frac{3}{4}$	$11\frac{1}{2}$	0.9799	0.1994	$\frac{1}{2}$
$\frac{1}{2}$	1.0000	.0087	$\frac{1}{2}$	$\frac{3}{4}$.9790	.2036	$\frac{1}{4}$
$\frac{3}{4}$.9999	.0131	$\frac{1}{4}$	12	.9781	.2079	78
1	.9998	.0175	89	$\frac{1}{4}$.9772	.2122	$\frac{3}{4}$
$\frac{1}{4}$.9998	.0218	$\frac{3}{4}$	$\frac{1}{2}$.9763	.2164	$\frac{1}{2}$
$\frac{1}{2}$.9997	.0262	$\frac{1}{2}$	$\frac{3}{4}$.9753	.2207	$\frac{1}{4}$
$\frac{3}{4}$.9995	.0305	$\frac{1}{4}$	13	.9744	.2250	77
2	.9994	.0349	88	$\frac{1}{4}$.9734	.2292	$\frac{3}{4}$
$\frac{1}{4}$.9992	.0393	$\frac{3}{4}$	$\frac{1}{2}$.9724	.2334	$\frac{1}{2}$
$\frac{1}{2}$.9990	.0436	$\frac{1}{2}$	$\frac{3}{4}$.9713	.2377	$\frac{1}{4}$
$\frac{3}{4}$.9988	.0480	$\frac{1}{4}$	14	.9703	.2419	76
3	.9986	.0523	87	$\frac{1}{4}$.9692	.2462	$\frac{3}{4}$
$\frac{1}{4}$.9984	.0567	$\frac{3}{4}$	$\frac{1}{2}$.9681	.2504	$\frac{1}{2}$
$\frac{1}{2}$.9981	.0610	$\frac{1}{2}$	$\frac{3}{4}$.9670	.2546	$\frac{1}{4}$
$\frac{3}{4}$.9979	.0654	$\frac{1}{4}$	15	.9659	.2588	75
4	.9976	.0698	86	$\frac{1}{4}$.9648	.2630	$\frac{3}{4}$
$\frac{1}{4}$.9973	.0741	$\frac{3}{4}$	$\frac{1}{2}$.9636	.2672	$\frac{1}{2}$
$\frac{1}{2}$.9969	.0785	$\frac{1}{2}$	$\frac{3}{4}$.9625	.2714	$\frac{1}{4}$
$\frac{3}{4}$.9966	.0828	$\frac{1}{4}$	16	.9613	.2756	74
5	.9962	.0872	85	$\frac{1}{4}$.9600	.2798	$\frac{3}{4}$
$\frac{1}{4}$.9958	.0915	$\frac{3}{4}$	$\frac{1}{2}$.9588	.2840	$\frac{1}{2}$
$\frac{1}{2}$.9954	.0958	$\frac{1}{2}$	$\frac{3}{4}$.9576	.2882	$\frac{1}{4}$
$\frac{3}{4}$.9950	.1002	$\frac{1}{4}$	17	.9563	.2924	73
6	.9945	.1045	84	$\frac{1}{4}$.9550	.2965	$\frac{3}{4}$
$\frac{1}{4}$.9941	.1089	$\frac{3}{4}$	$\frac{1}{2}$.9537	.3007	$\frac{1}{2}$
$\frac{1}{2}$.9936	.1132	$\frac{1}{2}$	$\frac{3}{4}$.9524	.3049	$\frac{1}{4}$
$\frac{3}{4}$.9931	.1175	$\frac{1}{4}$	18	.9511	.3090	72
7	.9925	.1219	83	$\frac{1}{4}$.9497	.3132	$\frac{3}{4}$
$\frac{1}{4}$.9920	.1262	$\frac{3}{4}$	$\frac{1}{2}$.9483	.3173	$\frac{1}{2}$
$\frac{1}{2}$.9914	.1305	$\frac{1}{2}$	$\frac{3}{4}$.9469	.3214	$\frac{1}{4}$
$\frac{3}{4}$.9909	.1349	$\frac{1}{4}$	19	.9455	.3256	71
8	.9903	.1392	82	$\frac{1}{4}$.9441	.3297	$\frac{3}{4}$
$\frac{1}{4}$.9897	.1435	$\frac{3}{4}$	$\frac{1}{2}$.9426	.3338	$\frac{1}{2}$
$\frac{1}{2}$.9890	.1478	$\frac{1}{2}$	$\frac{3}{4}$.9412	.3379	$\frac{1}{4}$
$\frac{3}{4}$.9884	.1521	$\frac{1}{4}$	20	.9397	.3420	70
9	.9877	.1564	81	$\frac{1}{4}$.9382	.3461	$\frac{3}{4}$
$\frac{1}{4}$.9870	.1607	$\frac{3}{4}$	$\frac{1}{2}$.9367	.3502	$\frac{1}{2}$
$\frac{1}{2}$.9863	.1650	$\frac{1}{2}$	$\frac{3}{4}$.9351	.3543	$\frac{1}{4}$
$\frac{3}{4}$.9856	.1694	$\frac{1}{4}$	21	.9336	.3584	69
10	.9848	.1736	80	$\frac{1}{4}$.9320	.3624	$\frac{3}{4}$
$\frac{1}{4}$.9840	.1779	$\frac{3}{4}$	$\frac{1}{2}$.9304	.3665	$\frac{1}{2}$
$\frac{1}{2}$.9833	.1822	$\frac{1}{2}$	$\frac{3}{4}$.9288	.3706	$\frac{1}{4}$
$\frac{3}{4}$.9825	.1865	$\frac{1}{4}$	22	.9272	.3746	68
11	.9816	.1908	79	$\frac{1}{4}$.9255	.3786	$\frac{3}{4}$
$\frac{1}{4}$.9808	.1951	$\frac{3}{4}$	$\frac{1}{2}$.9239	.3827	$\frac{1}{2}$
	Depart'e	Latitude	Bea'g		Depart'e	Latitude	Bea'g

TRAVERSE TABLE.

Bea'g	Latitude	Depart'e		Bea'g	Latitude	Depart'e	
0			0	0			0
$\frac{3}{4}$	0.9222	0.3867	$\frac{1}{4}$	34	0.8290	0.5592	56
23	.9205	.3907	67	$\frac{1}{4}$.8266	.5628	$\frac{3}{4}$
$\frac{1}{4}$.9188	.3947	$\frac{3}{4}$	$\frac{1}{2}$.8241	.5664	$\frac{1}{2}$
$\frac{1}{2}$.9171	.3987	$\frac{1}{2}$	$\frac{3}{4}$.8216	.5700	$\frac{1}{4}$
$\frac{3}{4}$.9153	.4027	$\frac{1}{4}$	35	.8192	.5736	55
24	.9135	.4067	66	$\frac{1}{4}$.8166	.5771	$\frac{3}{4}$
$\frac{1}{4}$.9118	.4107	$\frac{3}{4}$	$\frac{1}{2}$.8141	.5807	$\frac{1}{2}$
$\frac{1}{2}$.9100	.4147	$\frac{1}{2}$	$\frac{3}{4}$.8116	.5842	$\frac{1}{4}$
$\frac{3}{4}$.9081	.4187	$\frac{1}{4}$	36	.8090	.5878	54
25	.9063	.4226	65	$\frac{1}{4}$.8064	.5913	$\frac{3}{4}$
$\frac{1}{4}$.9045	.4266	$\frac{3}{4}$	$\frac{1}{2}$.8039	.5948	$\frac{1}{2}$
$\frac{1}{2}$.9026	.4305	$\frac{1}{2}$	$\frac{3}{4}$.8013	.5983	$\frac{1}{4}$
$\frac{3}{4}$.9007	.4344	$\frac{1}{4}$	37	.7986	.6018	53
26	.8988	.4384	64	$\frac{1}{4}$.7960	.6053	$\frac{3}{4}$
$\frac{1}{4}$.8969	.4423	$\frac{3}{4}$	$\frac{1}{2}$.7934	.6088	$\frac{1}{2}$
$\frac{1}{2}$.8949	.4462	$\frac{1}{2}$	$\frac{3}{4}$.7907	.6122	$\frac{1}{4}$
$\frac{3}{4}$.8930	.4501	$\frac{1}{4}$	38	.7880	.6157	52
27	.8910	.4540	63	$\frac{1}{4}$.7853	.6191	$\frac{3}{4}$
$\frac{1}{4}$.8890	.4579	$\frac{3}{4}$	$\frac{1}{2}$.7826	.6225	$\frac{1}{2}$
$\frac{1}{2}$.8870	.4617	$\frac{1}{2}$	$\frac{3}{4}$.7799	.6259	$\frac{1}{4}$
$\frac{3}{4}$.8850	.4656	$\frac{1}{4}$	39	.7771	.6293	51
28	.8829	.4695	62	$\frac{1}{4}$.7744	.6327	$\frac{3}{4}$
$\frac{1}{4}$.8809	.4733	$\frac{3}{4}$	$\frac{1}{2}$.7716	.6361	$\frac{1}{2}$
$\frac{1}{2}$.8788	.4772	$\frac{1}{2}$	$\frac{3}{4}$.7688	.6394	$\frac{1}{4}$
$\frac{3}{4}$.8767	.4810	$\frac{1}{4}$	40	.7660	.6428	50
29	.8746	.4848	61	$\frac{1}{4}$.7632	.6461	$\frac{3}{4}$
$\frac{1}{4}$.8725	.4886	$\frac{3}{4}$	$\frac{1}{2}$.7604	.6494	$\frac{1}{2}$
$\frac{1}{2}$.8704	.4924	$\frac{1}{2}$	$\frac{3}{4}$.7576	.6528	$\frac{1}{4}$
$\frac{3}{4}$.8682	.4962	$\frac{1}{4}$	41	.7547	.6561	49
30	.8660	.5000	60	$\frac{1}{4}$.7518	.6593	$\frac{3}{4}$
$\frac{1}{4}$.8638	.5038	$\frac{3}{4}$	$\frac{1}{2}$.7490	.6626	$\frac{1}{2}$
$\frac{1}{2}$.8616	.5075	$\frac{1}{2}$	$\frac{3}{4}$.7461	.6659	$\frac{1}{4}$
$\frac{3}{4}$.8594	.5113	$\frac{1}{4}$	42	.7431	.6691	48
31	.8572	.5150	59	$\frac{1}{4}$.7402	.6724	$\frac{3}{4}$
$\frac{1}{4}$.8549	.5188	$\frac{3}{4}$	$\frac{1}{2}$.7373	.6756	$\frac{1}{2}$
$\frac{1}{2}$.8526	.5225	$\frac{1}{2}$	$\frac{3}{4}$.7343	.6788	$\frac{1}{4}$
$\frac{3}{4}$.8504	.5262	$\frac{1}{4}$	43	.7314	.6820	47
32	.8480	.5299	58	$\frac{1}{4}$.7284	.6852	$\frac{3}{4}$
$\frac{1}{4}$.8457	.5336	$\frac{3}{4}$	$\frac{1}{2}$.7254	.6884	$\frac{1}{2}$
$\frac{1}{2}$.8434	.5373	$\frac{1}{2}$	$\frac{3}{4}$.7224	.6915	$\frac{1}{4}$
$\frac{3}{4}$.8410	.5410	$\frac{1}{4}$	44	.7193	.6947	46
33	.8387	.5446	57	$\frac{1}{4}$.7163	.6978	$\frac{3}{4}$
$\frac{1}{4}$.8363	.5483	$\frac{3}{4}$	$\frac{1}{2}$.7133	.7009	$\frac{1}{2}$
$\frac{1}{2}$.8339	.5519	$\frac{1}{2}$	$\frac{3}{4}$.7102	.7040	$\frac{1}{4}$
$\frac{3}{4}$.8315	.5556	$\frac{1}{4}$	45	.7071	.7071	45
	Depart'e	Latitude	Bea'g		Depart'e	Latitude	Bea'g

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